Announcements

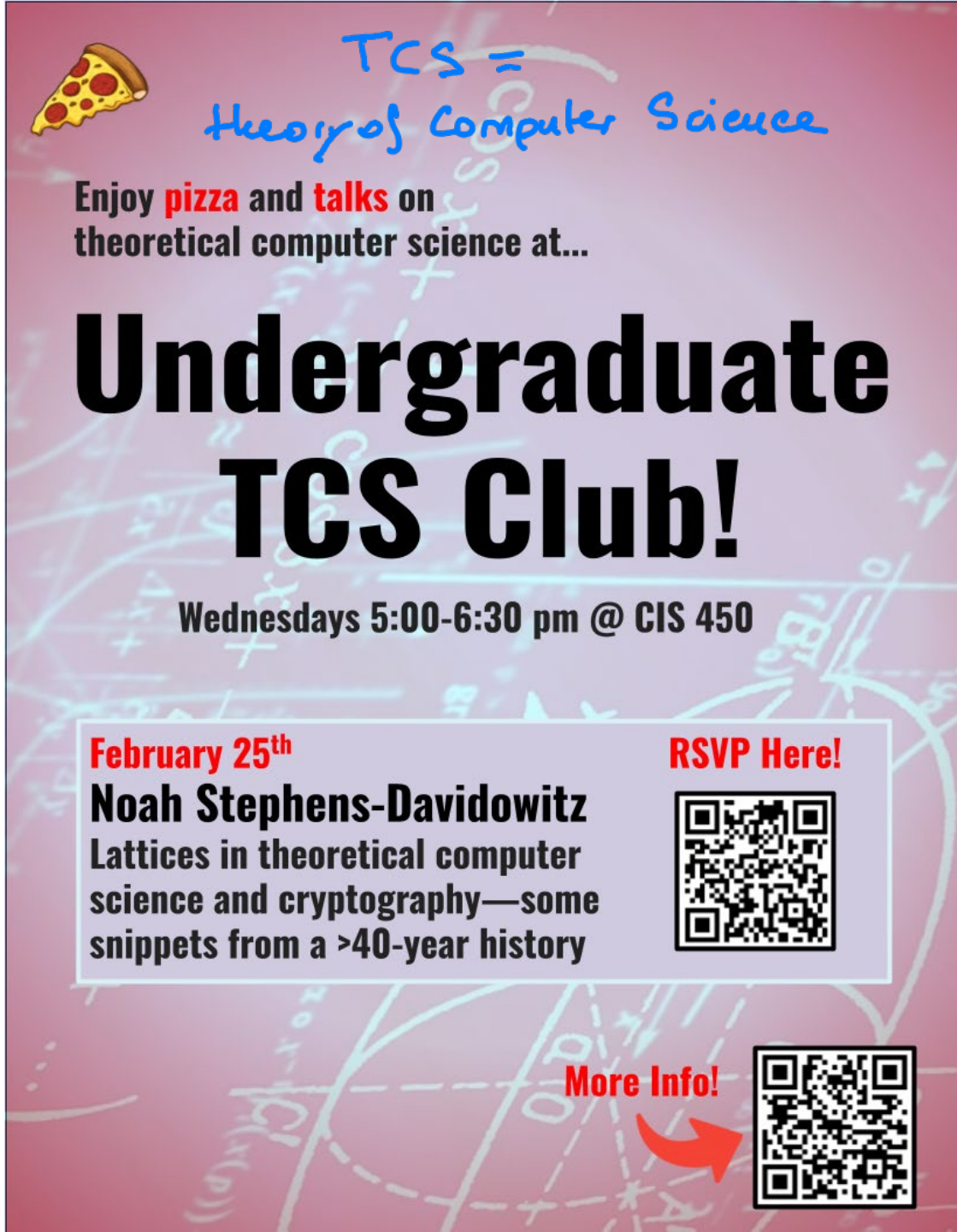
- Regrade requests are due a week after we open the grades. Today for the prelim

- All hw / exam questions
  prove all claims you make
  - algorithm correct
  - has claimed running time

- coding hw: only your last submission counts for credit

This week: applications of flows & cuts
today Friday

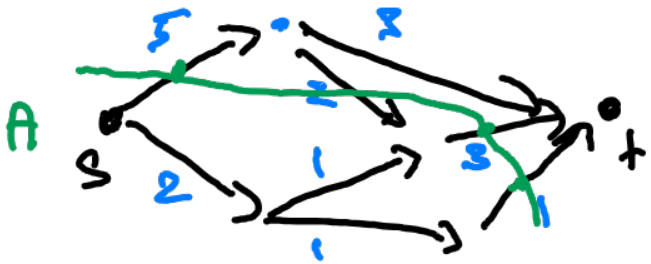
next week: start NP-completeness Ch 8



Today flow applications: demand/supply

Reminder flow network $G = (V, E)$

$c_e \geq 0$ on $e \in E$



wanted max value flow

$0 \leq f_e \leq c_e$ capacity $\forall e$

$\forall v \neq s, t$ flow in = flow out

value = 5 + 2

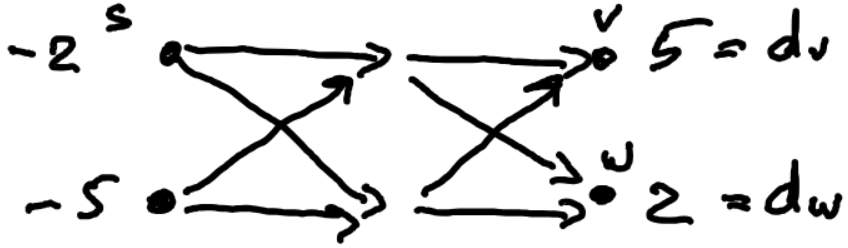
measured across cut

$5 + 3 + 1 - 2 = 7$

$A \ni s, \; t \notin A$ value(f) = flow out(A) − flow into(A)

$G = (V, E)$ directed graph

$c_e \geq 0$ capacities



d_v = demand at node v

$d_s < 0 \Rightarrow$ means supply

for now $\sum_v d_v = 0$

demand = supply

Question: is there a flow satisfying demands

$0 \leq f_e \leq c_e$ capacity constraint

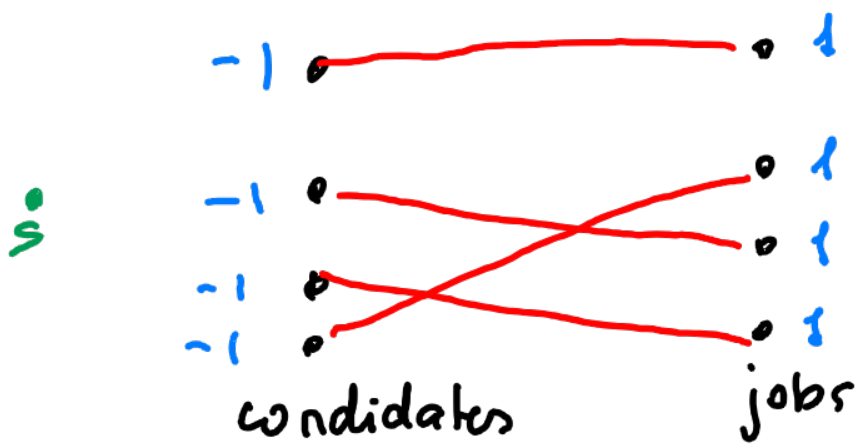
Today flow applications: demand/supply

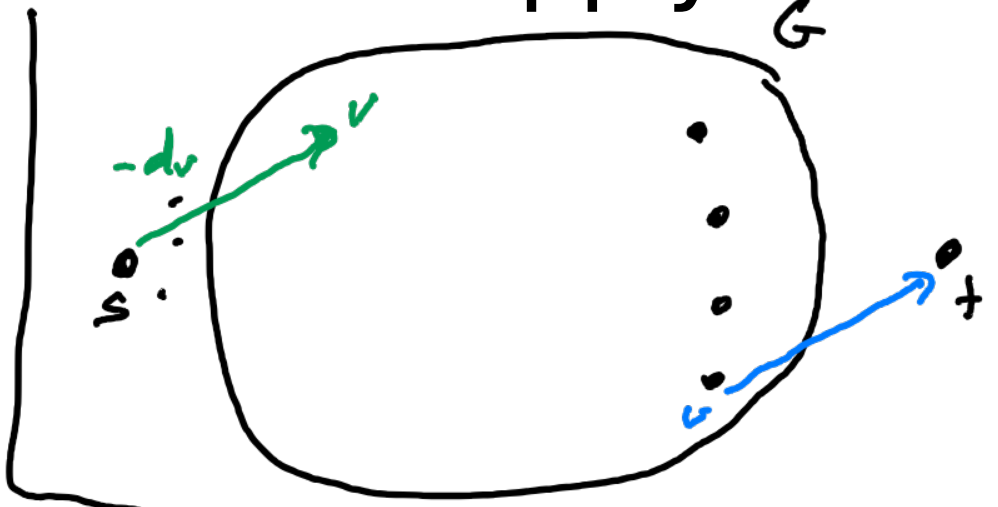
$\forall v \quad \sum_{(w,v)\in E} f_{wv} - \sum_{(v,w)\in E} f_{vw} = d_v$

flow in — flow out

Solve using flows

Example: perfect matchings





add extra s & t

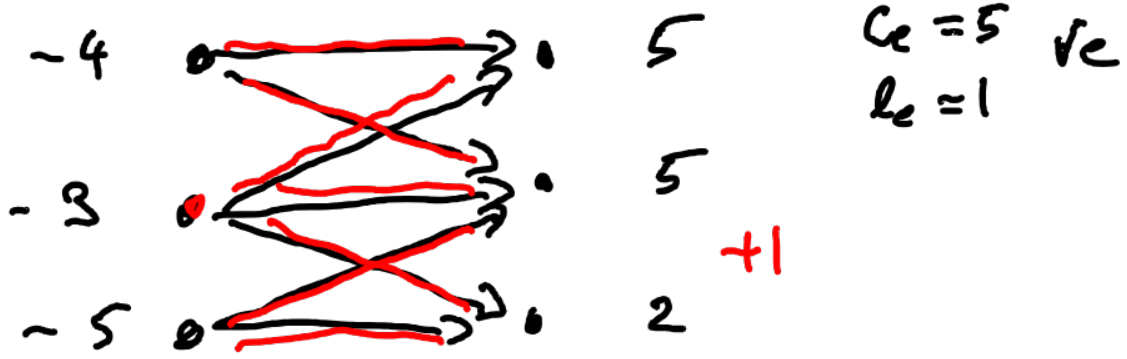
add (v,t) for $v \in V$ $d_v > 0$ capacity d_v

add (s,v) for $v \in V$ $d_v < 0$ capacity $-d_v$

Claim: flow satisfying demand exist if & only if max flow value $= \sum_{v: d_v > 0} d_v = -\sum_{v: d_v < 0} d_v$

proof sketch:
if $f_{sv} = c_{sv}$
$\Rightarrow$ v satisfies constraint

Demand/supply and lower bounds



$c_e = 5$ $\forall e$

$\ell_e = 1$

d_v supply / demand

edges $0 \le \ell_e \le c_e$ (ℓ_e: lower bound, c_e: capacity)

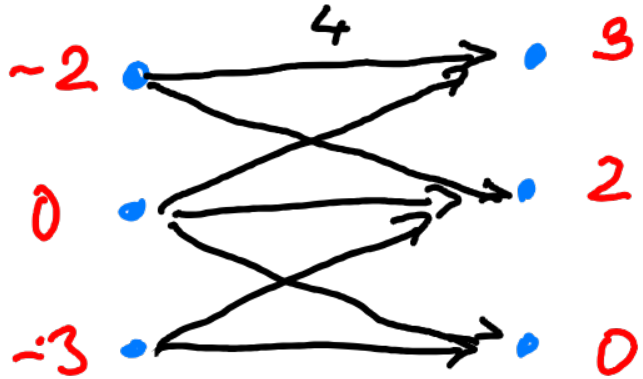
Given flow $\bar{f}_e$ satisfying demand/supply in residual problem

$\iff$ $f_e = \bar{f}_e + \ell_e$ flow for original problem

Can we reduce it to a solved problem
max flow or supply/demand

Idea: start $f_e = \ell_e$ all e

Residual problem



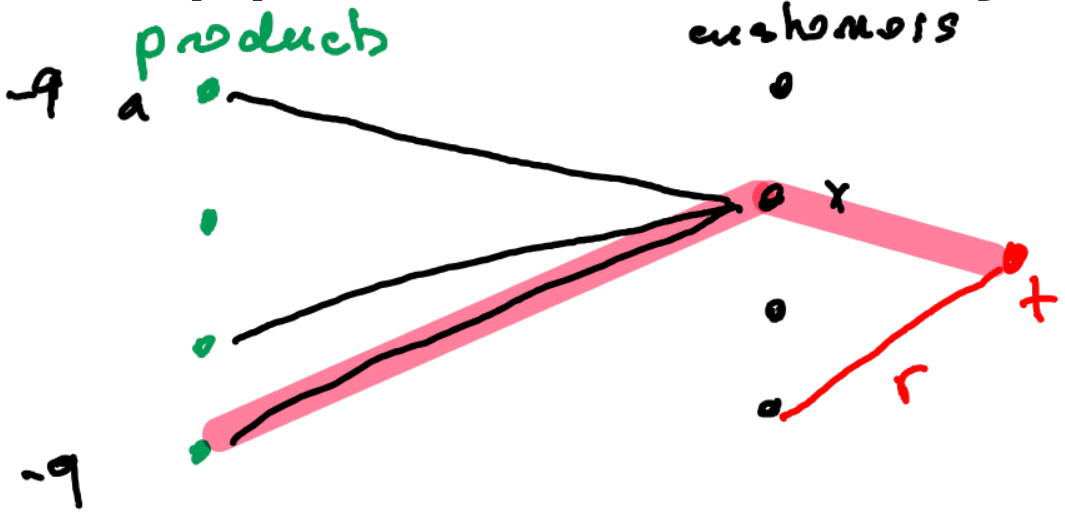
$\bar{c}_e = 4$ all edges

new capacity $\bar{c}_e = c_e - \ell_e$

extra allowed

$$\bar{d}_v = d_v - \left(\sum_{w:(w,v)\in E} \ell_{wv} - \sum_{w:(v,w)\in E} \ell_{vw} \right)$$

Application: Survey design



survey customers about some products they bought

edge (a, x) – x bought product a

– survey each product q times

– at most r questions / customers

product $d_v = -q$ supply

customers: add edges (x, t) capacity r

$d_v = 0$ for customers

$d_t = \#\text{products} \cdot q$

Poll question:

A: $d_t = 0$

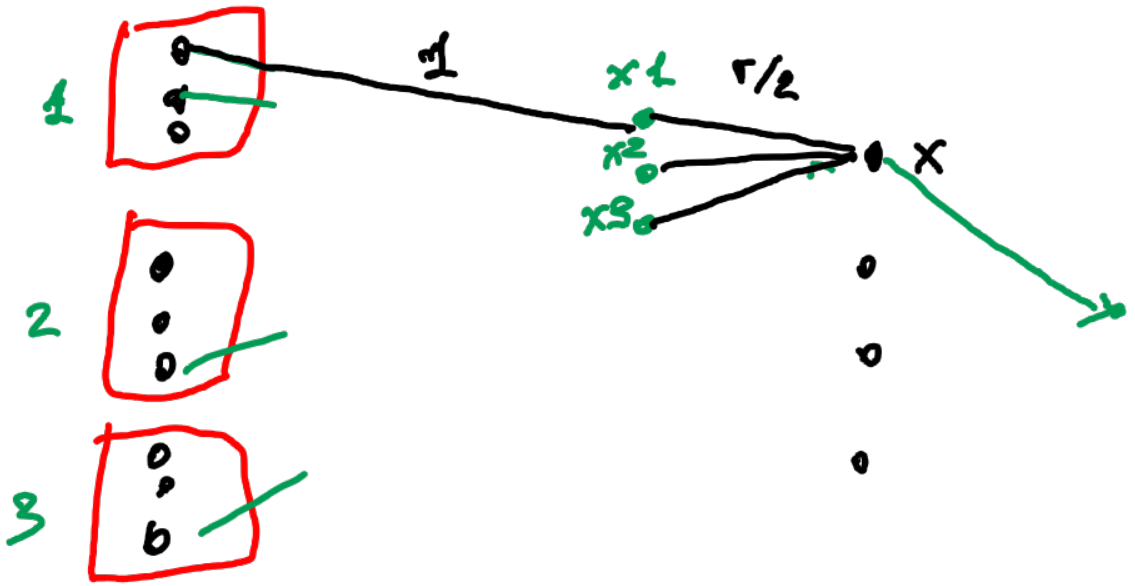
B: $d_t = q$

C: $d_t = \#\text{products}$

D: $d_t = \#\text{product} \cdot q$

Survey design take II

survey with groups of product



at most r question / customers
& at most r/2 for any one group

Idea: add new nodes

(xi) node for customer x for product type i

edges (xi, x) capacity r/2

edge (a, xi) if product of type i & x bought it . capacity 1

as before add edges (x,t) capacity r

demands −q for products

& q · # products = d_t

Claim: survey design satisfying rule exists if & only if flow satisfying demand exists.

Survey design take II

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